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Black hole entropy functions and attractor equations

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Abstract: The entropy and the attractor equations for static extremal black hole solutions follow from a variational principle based on an entropy function. In the general case such an entropy function can be derived from the reduced action evaluated in a near-horizon geometry. BPS black holes constitute special solutions of this variational principle, but they can also be derived directly from a different entropy function based on supersymmetry enhancement at the horizon. Both functions are consistent with electric/magnetic duality and for BPS black holes their corresponding OSV-type integrals give identical results at the semi-classical level. We clarify the relation between the two entropy functions and the corresponding attractor equations for $N = 2$ supergravity theories with higher-derivative couplings in four space-time dimensions. We discuss how non-holomorphic corrections will modify these entropy functions.

Keywords: Black Holes in String Theory, Black Holes.

Contents

1. Introduction

An important feature of (static) extremal black hole solutions is that scalar fields (often called moduli) tend to fixed values at the horizon determined by the black hole charges. These values are independent of the asymptotic values of the fields at spatial infinity. This fixed point behaviour is encoded in so-called attractor equations, which, in the generic case, can be understood from the field equations associated with the reduced action taken at a Killing horizon. The attractor equations are a crucial ingredient in comparing the macroscopic (or field-theoretic) black hole entropy with the microscopic (or statistical) entropy of a corresponding brane configuration. This and corresponding aspects of the relation between classical and quantum black holes have been studied extensively in the context of $N = 2$ supergravity in four space-time dimensions. Especially for BPS black holes many important results have been obtained. The inclusion of higher-derivative interactions into the effective actions often played a crucial role. For BPS black holes the attractor equations can be understood entirely from supersymmetry enhancement at the horizon. Obviously they must correspond to special solutions of the more general attractor equations based on a reduced action.

In this paper we study the relation between the more general attractor equations and the BPS attractor equations for static extremal black holes in four space-time dimensions. This can be done conveniently in terms of corresponding entropy functions that form the basis of an underlying variational principle. In the presence of higher-derivative actions it is very difficult to explicitly construct black hole solutions. However, by concentrating on the near-horizon region one can usually determine the fixed-point values directly without considering the interpolation between the horizon and spatial infinity. This approach was first applied to BPS black holes without higher-derivative interactions in [1 − 8] and then with higher-derivative interactions in [9 − 12]. It was also applied to non-BPS extremal black holes in [5, 6, 13 − 30]. In the presence of higher-derivative interactions full interpolating solutions have been studied for BPS black holes in [11, 31 − 33].

For $N = 2$ BPS black holes with higher-derivative interactions the attractor equations follow from classifying possible solutions with full supersymmetry [11]. As it turns out supersymmetry determines the near-horizon geometry (and thus the horizon area), the values of the moduli fields in terms of the charges and the value of the entropy as defined by the Noether charge definition of Wald [34]. For more general extremal black holes the analysis is more subtle and makes use of an action principle [13]. When dealing with spherically symmetric solutions, one can integrate out the spherical degrees of freedom and obtain a reduced action for a $1 + 1$ dimensional field theory. This action still describes the full black hole solutions. Under certain conditions the fixed values at the horizon can be obtained by considering the reduced action in a $1 + 1$ dimensional near-horizon geometry which has an enhanced symmetry (usually one has AdS_2). Near the horizon other fields respect this symmetry as well (when the enhanced symmetry is maximal the fields are all covariantly constant), so that the two-dimensional integral in the reduced action can be dropped and one obtains a potential depending on variables that specify the values of the fields at the Killing horizon. Actually the number of relevant variables can often be reduced already at an earlier stage by imposing some of the equations of motion at the level of the interpolating solution, but this represents no problem of principle.

This paper is organized as follows. In section 2 we consider the entropy function, both in the reduced action approach of [13] and in the context of BPS black holes (the latter for the case of $N = 2$ supergravity based on [7, 12]). We discuss those features that are relevant for electric/magnetic duality. In section 3 we evaluate the entropy function based on the action of a general $N = 2$ supergravity theory following [25], and we relate it to the BPS entropy function. We display the associated variational equations with and without higher-curvature interactions. For BPS black holes both entropy functions can be used in the definition of a corresponding duality invariant OSV-type integral and lead to identical results at the semi- classical level. In section 4 we briefly comment on corrections to the entropy functions due to other higher-derivative interactions associated with matter multiplets. We also discuss the modification of the entropy functions by non-holomorphic corrections.

2. Entropy functions

In this section we will briefly consider the entropy function derived from the action evaluated in a near-horizon geometry for some rather general theory and the entropy function that pertains to static BPS black holes in $N = 2$ supergravity in four space-time dimensions.

2.1 The reduced action and the entropy function

When considering spherically symmetric solutions one may integrate out the spherical degrees of freedom. This leads to a reduced action, which we consider here for a general system of abelian vector gauge fields, scalar and matter fields coupled to gravity. The geometry is then restricted to the product of the sphere S^2 and a 1+1 dimensional space-time, and the dependence of the fields on the S^2 coordinates θ and φ is fixed by symmetry arguments. For the moment we will not make any assumption regarding the dependence on the remaining two cooordinates r and t. Consequently we write the general field configuration consistent with the various isometries as

$$\begin{aligned} \mathrm{d}s^2{}_{(4)} &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}s^2{}_{(2)} + v_2\Big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\Big)\,, \\ F_{rt}{}^I &= e^I\,, \qquad F_{\theta\varphi}{}^I = \frac{p^I}{4\pi}\sin\theta\,. \end{aligned} \tag{2.1}$$

Here the $F_{\mu\nu}{}^I$ denote the field strengths associated with a number of abelian gauge fields. The θ-dependence of $F_{\theta\varphi}{}^I$ is fixed by rotational invariance and the p^I denote the magnetic charges. The latter are constant by virtue of the Bianchi identity, but all other fields are still functions of r and t. As we shall see in a moment the fields e^I are dual to the electric charges. The radius of S^2 is defined by the field v_2. The line element of the $1+1$ dimensional space-time will be expressed in terms of the two-dimensional metric $\bar g_{ij}$, whose determinant will be related to a field v_1 according to,

$$v_1 = \sqrt{|\bar g|}\,. \tag{2.2}$$

Eventually $\bar g_{ij}$ will be taken proportional to an AdS_2 metric,

$$\mathrm{d}s^2{}_{(2)} = \bar g_{ij}\,\mathrm{d}x^i\mathrm{d}x^j = v_1\Big(-r^2\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2}\Big)\,. \tag{2.3}$$

In addition to the fields e^I, v_1 and v_2 there may be a number of other fields which for the moment we denote collectively by u_α.

As is well known theories based on abelian vector fields are subject to electric/magnetic duality, because their equations of motion expressed in terms of the dual field strengths,[1]

$$G_{\mu\nu I} = \sqrt{|g|}\,\varepsilon_{\mu\nu\rho\sigma}\,\frac{\partial\mathcal{L}}{\partial F_{\rho\sigma}{}^I}\,, \tag{2.4}$$

take the same form as the Bianchi identities for the field strengths $F_{\mu\nu}{}^I$. Adopting the conventions where $x^\mu = (t,r,\theta,\varphi)$ and $\varepsilon_{tr\theta\varphi} = 1$, and the signature of the space-time metric equals $(-,+,+,+)$ as is obvious from (2.3), it follows that, in the background (2.1),

$$\begin{aligned} G_{\theta\varphi\,I} &= -v_1 v_2\,\sin\theta\,\frac{\partial\mathcal{L}}{\partial F_{rt}{}^I} = -v_1v_2\,\sin\theta\,\frac{\partial\mathcal{L}}{\partial e^I}\,, \\ G_{rt\,I} &= -v_1 v_2\,\sin\theta\,\frac{\partial\mathcal{L}}{\partial F_{\theta\varphi}{}^I} = -4\pi\,v_1v_2\,\frac{\partial\mathcal{L}}{\partial p^I}\,. \end{aligned} \tag{2.5}$$

[1] Here and henceforth we assume that the Lagrangian depends on the abelian field strengths but not on their space-time derivatives. This restriction is not an essential one. In case that the Lagrangian contains derivatives of field strengths, one replaces the derivative of the Lagrangian in (2.4) by the corresponding functional derivative of the action. We also assume that the gauge fields appear exclusively through their field strengths.

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These two tensors can be written as $q_I \sin\theta/(4\pi)$ and f_I. The quantities q_I and f_I are conjugate to p^I and e^I, respectively, and can be written as

$$
\begin{aligned}
q_I(e,p,v,u) &= -4\pi\, v_1 v_2\, \frac{\partial \mathcal{L}}{\partial e^I}\,, \\
f_I(e,p,v,u) &= -4\pi\, v_1 v_2\, \frac{\partial \mathcal{L}}{\partial p^I}\,.
\end{aligned} \tag{2.6}
$$

They depend on the constants p^I and on the fields e^I, $v_{1,2}$ and u_α, and possibly their t and r derivatives, but no longer on the S^2 coordinates θ and φ. Upon imposing the field equations it follows that the q_I are constant and correspond to the electric charges. Obviously our aim will be to obtain a description in terms of the charges p^I and q_I, rather than in terms of the p^I and e^I.

Electric/magnetic duality transformations are induced by rotating the tensors $F_{\mu\nu}{}^I$ and $G_{\mu\nu\, I}$ by a constant transformation, so that the new linear combinations are all subject to Bianchi identities. Half of them are then selected as the new field strengths defined in terms of new gauge fields, while the Bianchi identities on the remaining linear combinations are regarded as field equations belonging to a new Lagrangian defined in terms of the new field strengths. In order that this dualization can be effected the rotation betweeen the tensors must belong to $\mathrm{Sp}(2n+2;\mathbb{R})$, where $n+1$ denotes the number of independent gauge fields. Hence this leads to new quantities $(\tilde p^I, \tilde q_I)$ and $(\tilde e^I, \tilde f_I)$, where

$$
\begin{aligned}
\tilde p^I &= U^I{}_J\, p^J + Z^{IJ}\, q_J\,, \\
\tilde q_I &= V_I{}^J\, q_J + W_{IJ}\, p^J\,,
\end{aligned} \tag{2.7}
$$

and likewise for (e^I, f_I). Here $U^I{}_J$, $V_I{}^J$, W_{IJ} and Z^{IJ} are constant real $(n+1)\times(n+1)$ submatrices subject to

$$
\begin{aligned}
&U^{\mathrm{T}}V - W^{\mathrm{T}}Z = V^{\mathrm{T}}U - Z^{\mathrm{T}}W = \mathbb{1}\,, \\
&U^{\mathrm{T}}W = W^{\mathrm{T}}U\,, \qquad Z^{\mathrm{T}}V = V^{\mathrm{T}}Z\,,
\end{aligned} \tag{2.8}
$$

so that the full matrix belongs to $\mathrm{Sp}(2n+2;\mathbb{R})$ [35]. Since the charges are not continuous but will take values in an integer-valued lattice, this group should eventually be restricted to an appropriate arithmetic subgroup.

Subsequently we define the reduced Lagrangian by the integral of the full Lagrangian over S^2,

$$
\mathcal{F}(e,p,v,u) = \int \mathrm{d}\theta\, \mathrm{d}\varphi\, \sqrt{|g|}\, \mathcal{L}\,. \tag{2.9}
$$

We note that the definition of the conjugate quantities q_I and f_I takes the form,

$$
q_I = -\frac{\partial \mathcal{F}}{\partial e^I}\,, \qquad f_I = -\frac{\partial \mathcal{F}}{\partial p^I}\,. \tag{2.10}
$$

It is known that a Lagrangian does not transform as a function under electric/magnetic dualities. Instead we have [36],

$$
\tilde{\mathcal{F}}(\tilde e, \tilde p, v, u) + \frac{1}{2}[\tilde e^I \tilde q_I + \tilde f_I \tilde p^I] = \mathcal{F}(e,p,v,u) + \frac{1}{2}[e^I q_I + f_I p^I]\,. \tag{2.11}
$$

so that the linear combination $\mathcal{F}(e,p,v,u)+\frac{1}{2}[e^I q_I + f_I p^I]$ transforms as a function. Furthermore one may verify that first-order partial derivatives (say with respect to u or v, or derivatives thereof) of $\mathcal{F}(e,p,v,u)$ that leave e^I and p^I fixed, transform also as a function. This result implies that the field equations associated with fields other than the electromagnetic ones transform covariantly and retain their form when changing the electric/magnetic duality frame.

It is easy to see that the combination $e^I q_I - f_I p^I$ transforms as a function as well, so that we may construct a modification of (2.9) that no longer involves the f_I and that transforms as a function under electric/magnetic duality,

$$\mathcal{E}(q,p,v,u) = -\mathcal{F}(e,p,v,u) - e^I q_I \,, \tag{2.12}$$

which takes the form of a Legendre transform in view of the first equation (2.10). In this way we obtain a function of electric and magnetic charges. Therefore it transforms under electric/magnetic duality according to $\tilde{\mathcal{E}}(\tilde q,\tilde p,v,u) = \mathcal{E}(q,p,v,u)$. Furthermore the field equations imply that the q_I are constant and that the action, $\int \mathrm{d}t\mathrm{d}r\,\mathcal{E}$, is stationary under variations of the fields v and u, while keeping the p^I and q_I fixed. This is to be expected as $\mathcal{E}$ is in fact the analogue of the Hamiltonian density associated with the reduced Lagrangian density (2.9), at least as far as the vector fields are concerned.

In the near-horizon background (2.3), assuming fields that are invariant under the AdS_2 isometries, the generally covariant derivatives of the fields vanish and the equations of motion imply that the constant values of the fields $v_{1,2}$ and u_α are determined by demanding $\mathcal{E}$ to be stationary under variations of v and u,

$$\frac{\partial\mathcal{E}}{\partial v} = \frac{\partial\mathcal{E}}{\partial u} = 0\,, \qquad q_I = \text{constant}\,. \tag{2.13}$$

The function $2\pi\,\mathcal{E}(q,p,v,u)$ coincides with the entropy function proposed by Sen [13]. The first two equations of (2.13) are then interpreted as the attractor equations and the Wald entropy is directly proportional to the value of $\mathcal{E}$ at the stationary point,

$$\mathcal{S}_{\text{macro}}(p,q) \propto \mathcal{E}\Big|_{\text{attractor}}\,. \tag{2.14}$$

The normalization conventions used for the Lagrangian affect $\mathcal{E}$ and the definition of the charges and of Planck's constant. This has to be taken into account when determining the proportionality factor in (2.14), and we do so in (3.16). In the presentation above we followed the approach of [13], but similar approaches can be found in, for instance, [5, 6, 14]. Note that the entropy function does not necessarily depend on all fields at the horizon. The values of some of the fields will then be left unconstrained, but those will not appear in the expression for the Wald entropy.

The above derivation of the entropy function applies to any gauge and general coordinate invariant Lagrangian, and, in particular, also to Lagrangians containing higher-derivative interactions. In the absence of higher-derivative terms, the reduced Lagrangian $\mathcal{F}$ is at most quadratic in e^I and p^I and the Legendre transform (2.12) can easily be carried out. For instance, consider the following Lagrangian in four space-time dimensions (we only concentrate on terms quadratic in the field strengths),

$$\sqrt{|g|}\,\mathcal{L}_0 = -\frac{1}{4}\,\mathrm{i}\sqrt{|g|}\Big\{\mathcal{N}_{IJ}\,F^{+\,I}_{\mu\nu}\,F^{+\mu\nu J} - \bar{\mathcal{N}}_{IJ}\,F^{-\,I}_{\mu\nu}\,F^{-\mu\nu J}\Big\}\,, \tag{2.15}$$

where $F^{\pm\,I}_{\mu\nu}$ denote the (anti)-selfdual field strengths. In the context of this paper the tensors $F^{\pm I}_{\underline{rt}} = \pm\mathrm{i}F^{\pm\,I}_{\underline{\theta\varphi}} = \frac{1}{2}(F_{\underline{rt}}{}^I \pm \mathrm{i}F_{\underline{\theta\varphi}}{}^I)$ are relevant, where underlined indices refer to the tangent space. From (2.15), (2.1) and (2.3), we straightforwardly derive the associated reduced Lagrangian (2.9),

$$\mathcal{F} = \frac{1}{4}\left\{\frac{\mathrm{i}v_1\,p^I(\bar{\mathcal{N}}-\mathcal{N})_{IJ}\,p^J}{4\pi\,v_2} - \frac{4\mathrm{i}\pi\,v_2\,e^I(\bar{\mathcal{N}}-\mathcal{N})_{IJ}\,e^J}{v_1}\right\} - \frac{1}{2}e^I(\mathcal{N}+\bar{\mathcal{N}})_{IJ}\,p^J\;. \tag{2.16}$$

It is straightforward to evaluate the entropy function (2.12) in this case,

$$\mathcal{E} = -\frac{v_1}{8\pi\,v_2}\,(q_I - \mathcal{N}_{IK}\,p^K)\,[(\mathrm{Im}\,\mathcal{N})^{-1}]^{IJ}\,(q_J - \bar{\mathcal{N}}_{JL}\,p^L)\,, \tag{2.17}$$

which is indeed compatible with electric/magnetic duality. Upon decomposing into real matrices, $\mathrm{i}\mathcal{N}_{IJ} = \mu_{IJ} - \mathrm{i}\nu_{IJ}$, this result coincides with the corresponding terms in the so-called black hole potential discussed in [5, 6], and, more recently, in [14].

2.2 The BPS entropy function

In the previous subsection the symmetry of the near-horizon geometry played a crucial role. For BPS black holes the supersymmetry enhancement at the horizon is the crucial input that constrains certain fields at the horizon as well as the near-horizon geometry. Unlike in the previous case, the number of attractor equations is clear and is in principle given by the number of independent supermultiplets. However, the precise nature of these constraints is not always a priori clear. For instance, in the case of $N = 2$ supergravity, which we will be dealing with in more detail in subsequent sections, the requirement of supersymmetry enhancement allows the hypermultiplet scalars to take arbitrary values, while the value of the vector multiplet scalars is constrained by the black hole charges.

The $N = 2$ vector multiplets contain complex physical scalar fields which we denote by X^I. In supergravity these fields are defined projectively. At the two-derivative-level, the action for the vector multiplets is encoded in a holomorphic function $F(X)$. The coupling to supergravity requires this function to be homogeneous of second degree. Here we follow the conventions of [11], where the charges and the Lagrangian have different normalizations than in the previous subsection. However this subsection and the previous one are self-contained, and the issue of relative normalizations will only play a role in section 3. There is one issue, however, that needs to be discussed. In principle electric/magnetic duality is a feature that pertains to the gauge fields. Straightforward application of such a duality to an $N = 2$ supersymmetric Lagrangian with vector multiplets, leads to a new Lagrangian that no longer takes the canonical form in terms of a function $F(X)$. In order to bring it into that form one must simultaneously apply a field redefinition to the scalar and spinor fields. On the scalar fields, this redefinition follows from the observation that $(X^I, F_I(X))$ transforms as a sympletic vector analogous to the tensors $(F_{\mu\nu}{}^I, G_{\mu\nu I})$ discussed previously. The need

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for this field redefinition clearly follows from the observation that the gauge fields and the fields X^I have a well-defined relation imposed by supersymmetry. When integrating the rotated version of the F_I one obtains the new function $\tilde{F}(\tilde{X})$ in terms of which the new Lagrangian is encoded. Therefore, in the following, the duality relation of $(X^I, F_I(X))$ will have to be taken into account. We refer to [36, 37] for further details and a convenient list of formulae.

Upon a suitable uniform field-dependent rescaling of the fields, the BPS attractor equations take a convenient form[2] which is manifestly consistent with electric/magnetic duality,

$$\mathcal{P}^I = 0\,, \qquad \mathcal{Q}_I = 0\,, \qquad \Upsilon = -64\,, \tag{2.18}$$

where

$$\begin{aligned} \mathcal{P}^I &\equiv p^I + \mathrm{i}(Y^I - \bar{Y}^I)\,, \\ \mathcal{Q}_I &\equiv q_I + \mathrm{i}(F_I - \bar{F}_I)\,. \end{aligned} \tag{2.19}$$

Here the Y^I are related to the X^I by the uniform rescaling and F_I denotes the derivative of $F(Y)$ with respect to Y^I. Furthermore Υ is a complex scalar field equal to the square of the $N = 2$ auxiliary field $T_{ab}{}^{ij}$ of the Weyl multiplet (upon the uniform rescaling), which is an anti-selfdual Lorentz tensor. Note that for fields satisfying the attractor equations (2.18), one easily establishes that

$$|Z|^2 \equiv p^I F_I - q_I Y^I\,, \tag{2.20}$$

is equal to $\mathrm{i}(\bar{Y}^I F_I - Y^I \bar{F}_I)$ and therefore real; Z is sometimes refered to as the 'holomorphic BPS mass' and equals the central charge for the vector supermultiplet system. In terms of the original variables X^I it is defined as

$$Z = \exp[\mathcal{K}/2]\,(p^I F_I(X) - q_I X^I)\,, \tag{2.21}$$

where

$$\mathrm{e}^{-\mathcal{K}} = \mathrm{i}\,(\bar{X}^I F_I(X) - \bar{F}_I(\bar{X}) X^I)\,. \tag{2.22}$$

At the horizon the variables Y^I are defined by

$$Y^I = \exp[\mathcal{K}/2]\,\bar{Z}\,X^I\,. \tag{2.23}$$

It is possible to incorporate higher-order derivative interactions involving the square of the Weyl tensor, by including the Weyl multiplet into the function F, preserving its homogeneity according to

$$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2\, F(Y, \Upsilon)\,. \tag{2.24}$$

As it turns out [11] this modification does not change the form of the attractor equations (2.18).

The BPS attractor equations can also be described by a variational principle based on an entropy function [7, 12],

$$\Sigma(Y, \bar{Y}, p, q) = \mathcal{F}(Y, \bar{Y}, \Upsilon, \bar{\Upsilon}) - q_I(Y^I + \bar{Y}^I) + p^I(F_I + \bar{F}_I)\,, \tag{2.25}$$

[2] We ignore the hypermultiplets at this stage.

where p^I and q_I couple to the corresponding magneto- and electrostatic potentials at the horizon (cf. [11]) in a way that is consistent with electric/magnetic duality. The quantity $\mathcal{F}(Y,\bar{Y},\Upsilon,\bar{\Upsilon})$, which will be denoted as the free energy, is defined by

$$\mathcal{F}(Y,\bar{Y},\Upsilon,\bar{\Upsilon}) = -\mathrm{i}\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) - 2\mathrm{i}\left(\Upsilon F_\Upsilon - \bar{\Upsilon}\bar{F}_\Upsilon\right) , \tag{2.26}$$

where $F_\Upsilon = \partial F/\partial\Upsilon$. Also this expression is compatible with electric/magnetic duality [37]. Varying the entropy function Σ with respect to the Y^I, while keeping the charges and Υ fixed, yields the result,

$$\delta\Sigma = \mathcal{P}^I\,\delta(F_I + \bar{F}_I) - \mathcal{Q}_I\,\delta(Y^I + \bar{Y}^I) . \tag{2.27}$$

Here we made use of the homogeneity of the function $F(Y,\Upsilon)$. Under the mild assumption that the matrix

$$N_{IJ} = \mathrm{i}(\bar{F}_{IJ} - F_{IJ}), \tag{2.28}$$

is non-degenerate, it thus follows that stationary points of Σ satisfy the attractor equations. The macroscopic entropy is equal to the entropy function taken at the attractor point. This implies that the macroscopic entropy is the Legendre transform of the free energy $\mathcal{F}$. An explicit calculation yields the entropy formula [9],

$$\mathcal{S}_{\mathrm{macro}}(p,q) = \pi\,\Sigma\Big|_{\mathrm{attractor}} = \pi\Big[|Z|^2 - 256\,\mathrm{Im}\,F_\Upsilon\Big]_{\Upsilon=-64} . \tag{2.29}$$

Here the first term represents a quarter of the horizon area (in Planck units) so that the second term defines the deviation from the Bekenstein-Hawking area law. In view of the homogeneity properties and the fact that Υ takes a fixed value the second term will be subleading in the limit of large charges. Note, however, that also the area will contain subleading terms, as it will also depend on Υ. In the absence of Υ-dependent terms, the homogeneity of the function $F(Y)$ implies that the area scales quadratically with the charges.

We should emphasize that also other higher-derivative interactions can be present and those will not be captured by the function $F(Y,\Upsilon)$. We will return to this issue in section 4.

3. Application to N=2 supergravity

We now study the various entropy functions for $N=2$ supergravity systems. Following [25] we will first determine the form of the entropy function $\mathcal{E}$. Subsequently we will exhibit its relation to the BPS entropy function Σ. The supergravity Lagrangian consists of various parts. The most important one concerns the vector multiplets, including the possible effect from the Weyl multiplet. To this we have to add the Lagrangian for a second compensating supermultiplet, which we take to be a hypermultiplet. Other choices for the compensating multiplet (three different choices have been studied in the literature [38]) are, of course, possible and should yield identical results. Additional hypermultiplets may also be added,

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but play a passive role in the following. The relevant Lagrangian is given by [11],

$$
\begin{aligned}
8\pi\, e^{-1}\,\mathcal{L} = {}& \mathrm{i}\mathcal{D}^{\mu}F_I\,\mathcal{D}_{\mu}\bar{X}^I - \mathrm{i}F_I\,\bar{X}^I\left(\frac{1}{6}R - D\right) - \frac{1}{8}\mathrm{i}F_{IJ}\,Y_{ij}{}^{I}\,Y^{Jij} - \frac{1}{4}\mathrm{i}\hat{B}_{ij}\,F_{AI}Y^{Iij} \\
& + \frac{1}{4}\mathrm{i}F_{IJ}\left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T_{ab}{}^{ij}\varepsilon_{ij}\right)\left(F^{-Jab} - \frac{1}{4}\bar{X}^J T^{ijab}\varepsilon_{ij}\right) \\
& - \frac{1}{8}\mathrm{i}F_I\left(F^{+I}_{ab} - \frac{1}{4}X^I T_{abij}\varepsilon^{ij}\right)T^{ab}{}_{ij}\varepsilon^{ij} + \frac{1}{2}\mathrm{i}\hat{F}^{-ab}\,F_{AI}\left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T_{ab}{}^{ij}\varepsilon_{ij}\right) \\
& + \frac{1}{2}\mathrm{i}F_A\hat{C} - \frac{1}{8}\mathrm{i}F_{AA}(\varepsilon^{ik}\varepsilon^{jl}\hat{B}_{ij}\hat{B}_{kl} - 2\hat{F}^{-}_{ab}\hat{F}^{-ab}) - \frac{1}{32}\mathrm{i}F(T_{abij}\varepsilon^{ij})^2 + \text{h.c.} \\
& - \frac{1}{2}\varepsilon^{ij}\,\bar{\Omega}_{\alpha\beta}\,\mathcal{D}_{\mu}A_i{}^{\alpha}\,\mathcal{D}^{\mu}A_j{}^{\beta} + \chi\left(\frac{1}{6}R + \frac{1}{2}D\right), \qquad (3.1)
\end{aligned}
$$

where the last two terms pertain to the hypermultiplets. This expression is consistent with electric/magnetic duality upon use of the field equations for the vector fields and the auxiliary fields $Y_{ij}{}^{I}$ [37]. The quantities $A_i{}^{\alpha}(\phi)$ denote the hypermultiplet sections, and χ denotes the hyper-Kähler potential. We refrain from giving explicit definitions at this point and refer the reader to [39]. The covariant derivatives involve all the bosonic gauge fields, such as the Lorentz spin connection and the gauge fields associated with Weyl rescalings and the $\mathrm{SU}(2)\times\mathrm{U}(1)$ R-symmetry. The quantities X^I, $F^{\pm I}_{ab}$ and $Y_{ij}{}^{I}$ denote the bosonic components of the vector multiplets, namely, the complex scalars, the (anti-)selfdual field strengths (defined with tangent-space indices) and the auxiliary fields, respectively. As we already explained, the anti-selfdual tensor field $T_{ab}{}^{ij}$ belongs to the Weyl multiplet and defines the lowest component of a scalar chiral multiplet, $\hat{A} = (T_{ab}{}^{ij}\varepsilon_{ij})^2$, which, upon rescaling yields the field Υ introduced earlier. Apart from $T_{ab}{}^{ij}$, the bosonic components of the Weyl multiplet comprise the Riemann curvature, the field strengths of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge fields associated with R-symmetry, and a real scalar field denoted by D. The quantities $\hat{B}_{ij}$, $\hat{F}^{\pm}_{ab}$ and $\hat{C}$ denote the other bosonic components of the scalar chiral multiplet constructed from the Weyl multiplet. For the exact expressions we refer to [11].

The fact that we extracted a uniform factor of 8π from the Lagrangian and the fact that the charges used in [11] differ from the charges introduced in (2.1) and in (2.6), implies that the charges p^I and q_I as defined in subsection 2.1 should be changed according to: $p^I \to 4\pi\,p^I$ and $q_I \to \frac{1}{2}q_I$. This rescaling has been carried out in all subsequent formulae.

The next step is to exploit the spherical symmetry and derive the reduced Lagrangian (2.9). For the space-time metric and the field strengths this was already done in (2.1). Let us first concentrate on the auxiliary field $T_{ab}{}^{ij}$, which plays an important role in this paper. In a spherically symmetric configuration this field can be expressed in terms of a single complex scalar w. Following [25] we define,

$$
T_{\underline{rt}}{}^{ij}\varepsilon_{ij} = -\mathrm{i}\,T_{\underline{\theta\varphi}}{}^{ij}\varepsilon_{ij} = w\,, \qquad (3.2)
$$

where underlined indices denote tangent-space indices. Consequently we have $\hat{A} = -4w^2$. We will have to do the same for all other fields, but we will restrict ourselves to a restricted class of solutions by putting some of the fields to zero. Namely, at this stage we will assume the following consistent set of constraints,

$$R(\mathcal{V})_{\mu\nu}{}^{i}{}_{j} = R(A)_{\mu\nu} = \mathcal{D}_\mu X^I = \mathcal{D}_\mu A_i{}^\alpha = 0\,, \tag{3.3}$$

where the first two tensors denote the R-symmetry field strengths. These constraints are weaker than the ones imposed in [25], and they are in accord with those that follow from requiring supersymmetry enhancement at the horizon [11]. It is not unlikely that, if one were to relax these constraints in the evaluation of the reduced Lagrangian, most of them would still emerge in the form of attractor equations at the end. We will not pursue this question in any detail.

Since $\hat{B}_{ij}$ is proportional to $R(\mathcal{V})_{\mu\nu}{}^{i}{}_{j}$, this field can thus be ignored as well. Furthermore the auxiliary fields $Y_{ij}{}^I$ can be dropped as a result of their equations of motion. Subject to all these conditions the relevant expressions for $\hat{C}$ and $\hat{F}_{\mu\nu}$ are as follows,

$$\begin{aligned}
\hat{F}^{-ab} &= -16\,\mathcal{R}(M)_{cd}{}^{ab}\,T^{klcd}\,\varepsilon_{kl}\,,\\
\hat{C} &= 64\,\mathcal{R}(M)^{-cd}{}_{ab}\,\mathcal{R}(M)^{-ab}_{cd} - 32\,T^{ab\,ij}\,D_a\,D^c T_{cb\,ij}\,,
\end{aligned} \tag{3.4}$$

where $\mathcal{R}$ is a modification of the Riemann tensor and the derivatives are superconformally invariant [11]. Under the same assumptions the Lagrangian (3.1) reduces to $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$, with

$$\begin{aligned}
8\pi\, e^{-1}\,\mathcal{L}_1 =&\ \Big[\frac{1}{4}\mathrm{i}F_{IJ}F^{-I}_{ab}\Big(F^{-Jab} - \frac{1}{2}\bar{X}^J T^{ijab}\varepsilon_{ij}\Big)\\
&\quad -\frac{1}{8}\mathrm{i}F_I\,F^{+I}_{ab}\,T^{ab}{}_{ij}\varepsilon^{ij} + \frac{1}{2}\mathrm{i}\hat{F}^{-ab}\,F_{AI}\,F^{-I}_{ab} + \text{h.c.}\Big]\,,\\
8\pi\, e^{-1}\,\mathcal{L}_2 =&\ \mathrm{e}^{-\mathcal{K}}\Big(D - \frac{1}{6}R\Big) + \frac{1}{2}\chi\Big(D + \frac{1}{3}R\Big)\\
&\quad -\frac{1}{32}\Big[\mathrm{i}\Big(F - F_I X^I + \frac{1}{2}\bar{F}_{IJ}X^I X^J\Big)(T_{abij}\varepsilon^{ij})^2 + \text{h.c.}\Big]\\
&\quad +\frac{1}{2}\Big[\mathrm{i}F_A\hat{C} + \frac{1}{2}\mathrm{i}F_{AA}\,\hat{F}^-_{ab}\hat{F}^{-ab} - \frac{1}{4}\mathrm{i}\hat{F}^{-ab}\,F_{AI}\bar{X}^I T_{ab}{}^{ij}\varepsilon_{ij} + \text{h.c.}\Big]\,.
\end{aligned} \tag{3.5}$$

In the AdS_2 background we are left with a restricted number of field variables that are all constant, namely, v_1, v_2, w, D, e^I, X^I and $A_i{}^\alpha$. Note, however, that the dependence on the fields $A_i{}^\alpha$ is entirely contained in the hyperkähler potential χ. Our next task is to evaluate the reduced Lagrangian as a function of these variables. Before doing so, we should stress that the above Lagrangian (3.1) was derived from a superconformally invariant expression. As a result the bosonic quantities are still subject to certain invariance transformations. One of them is scale invariance with respect to a complex parameter λ,

$$v_{1,2} \to |\lambda|^{-2} v_{1,2}\,,\quad w \to \bar{\lambda} w\,,\quad D \to |\lambda|^2 D\,,\quad X^I \to \bar{\lambda} X^I\,,\quad \chi \to |\lambda|^2\chi\,. \tag{3.6}$$

All other fields (as well as the charges) are invariant under these scale transformation. In addition the hypermultiplet sections are subject to rigid SU(2) transformations. The reduced Lagrangian and the entropy function should be invariant under these transformations. Therefore it will be useful to express the entropy function (2.12) computed from the Lagrangian (3.1) in terms of a set of scale invariant variables. We choose the following set of such variables,

$$
\begin{aligned}
& Y^I = \frac{1}{4} v_2\, \bar{w}\, X^I\,, \quad \Upsilon = \frac{1}{16} v_2^2\, \bar{w}^2\, \hat{A} = -\frac{1}{4} v_2^2\, |w|^4\,, \quad U = \frac{v_1}{v_2}\,, \\
& \tilde{D} = v_2\, D + \frac{2}{3}(U^{-1} - 1)\,, \quad \tilde{\chi} = v_2\, \chi\,.
\end{aligned} \tag{3.7}
$$

Observe that Υ is real and negative, and that $\sqrt{-\Upsilon}$ and U are real and positive. Note also that the hypermultiplets contribute only through the hyperkähler potential χ.

We now compute the quantities appearing in (3.5) for the near-horizon background specified in terms of the parameters given above. We obtain (indices i, j refer to the AdS_2 coordinates r, t, whereas indices α, β refer to S^2 coordinates θ, φ),

$$
\begin{aligned}
R &= 2\left(v_1^{-1} - v_2^{-1}\right)\,, \\
f_i{}^j &= \left[\frac{1}{2} v_1^{-1} - \frac{1}{4}\left(D + \frac{1}{3}R\right) - \frac{1}{32}|w|^2\right] \delta_i{}^j\,, \\
f_\alpha{}^\beta &= \left[-\frac{1}{2} v_2^{-1} - \frac{1}{4}\left(D + \frac{1}{3}R\right) + \frac{1}{32}|w|^2\right] \delta_\alpha{}^\beta\,, \\
\mathcal{R}(M)_{ij}{}^{kl} &= \left(D + \frac{1}{3}R\right) \delta_{ij}{}^{kl}\,, \\
\mathcal{R}(M)_{\alpha\beta}{}^{\gamma\delta} &= \left(D + \frac{1}{3}R\right) \delta_{\alpha\beta}{}^{\gamma\delta}\,, \\
\mathcal{R}(M)_{i\alpha}{}^{j\beta} &= \frac{1}{2}\left(D - \frac{1}{6}R\right) \delta_i^j\, \delta_\alpha^\beta\,, \\
\hat{A} &= -4w^2\,, \\
\hat{F}^-_{\underline{rt}} &= -\mathrm{i}\hat{F}^-_{\underline{\theta\varphi}} = -16w\left(D + \frac{1}{3}R\right)\,, \\
\hat{C} &= 192 D^2 + \frac{32}{3}R^2 - 16|w|^2(v_1^{-1} + v_2^{-1}) + 2|w|^4\,.
\end{aligned} \tag{3.8}
$$

With these results we obtain the following contributions to the reduced Lagrangian corresponding to $\mathcal{L}_1$ and $\mathcal{L}_2$ of (3.5),

$$
\begin{aligned}
\mathcal{F}_1 = {} & \frac{1}{8} N_{IJ}\Big[U^{-1} e^I e^J - U p^I p^J\Big] - \frac{1}{4}(F_{IJ} + \bar{F}_{IJ}) e^I p^J \\
& + \frac{1}{2}\mathrm{i} e^I \Big[F_I + F_{IJ}\bar{Y}^J + 8 F_{I\Upsilon}\sqrt{-\Upsilon}\,\tilde{D} - \text{h.c.}\Big] \\
& - \frac{1}{2} U p^I \Big[F_I - F_{IJ}\bar{Y}^J - 8 F_{I\Upsilon}\sqrt{-\Upsilon}\,\tilde{D} + \text{h.c.}\Big]\,, \\
\mathcal{F}_2 = {} & \frac{4\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I)(\tilde{D}U + U - 1) + \frac{1}{4}\tilde{\chi}\tilde{D}U \\
& + \mathrm{i}U\Big[F - Y^I F_I - 2\Upsilon F_\Upsilon + \frac{1}{2}\bar{F}_{IJ} Y^I Y^J - \text{h.c.}\Big] \\
& + \mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\Big[48 U\tilde{D}^2 + 64\tilde{D}(U - 1) + 32(U + U^{-1} - 2) - 8(1 + U)\sqrt{-\Upsilon}\Big] \\
& + 32\mathrm{i}U\Big[\tilde{D}^2\Upsilon F_{\Upsilon\Upsilon} - \frac{1}{4}\tilde{D}\,\bar{Y}^I F_{I\Upsilon}\sqrt{-\Upsilon} - \text{h.c.}\Big]\,.
\end{aligned} \tag{3.9}
$$

Observe that these results refer to a general function $F(Y,\Upsilon)$. Because of the scale invariance, there is no longer a dependence on the field w. Furthermore we used the definition (2.28).

The entropy function can be written as

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 \,, \tag{3.10}$$

where $\mathcal{E}_1 = -\mathcal{F}_1 - \frac{1}{2}e^I q_I$ and $\mathcal{E}_2 = -\mathcal{F}_2$. Note that the factor 1/2 in $\mathcal{E}_1$ is due to the rescaling discussed earlier. When expressed in terms of p^I and q_I, $\mathcal{E}_1$ reads,

$$\begin{aligned} \mathcal{E}_1 =& \frac{1}{2}U\,\Sigma(Y,\bar{Y},p,q) + \frac{1}{2}U\,N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L) \\ &+\mathrm{i}U\Big[\Upsilon F_\Upsilon - \frac{1}{2}Y^I F_I + \frac{1}{2}\bar{F}_{IJ}Y^I Y^J - \text{h.c.}\Big] \\ &+8\mathrm{i}U\tilde{D}\sqrt{-\Upsilon}\Big[F_{I\Upsilon}N^{IJ}(\mathcal{Q}_J - \bar{F}_{JK}\mathcal{P}^K) - \text{h.c.}\Big] \\ &-8\mathrm{i}U\tilde{D}\sqrt{-\Upsilon}\Big[\bar{Y}^I F_{I\Upsilon} - \text{h.c.}\Big] \\ &+32U\,\tilde{D}^2\Upsilon\,N^{IJ}(F_{I\Upsilon} - \bar{F}_{I\Upsilon})(F_{J\Upsilon} - \bar{F}_{J\Upsilon})\,, \end{aligned} \tag{3.11}$$

where $\mathcal{Q}_I$, $\mathcal{P}^I$, and Σ were defined already in (2.19) and (2.25), respectively. Combining this result with $\mathcal{E}_2$ there are some crucial rearrangements and the result is an entropy function that is consistent with electric/magnetic duality,

$$\begin{aligned} \mathcal{E} =& \frac{1}{2}U\,\Sigma(Y,\bar{Y},p,q) + \frac{1}{2}U\,N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L) \\ &+8\mathrm{i}U\tilde{D}\sqrt{-\Upsilon}\Big[F_{I\Upsilon}N^{IJ}(\mathcal{Q}_J - \bar{F}_{JK}\mathcal{P}^K) - \text{h.c.}\Big] \\ &-\frac{4\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I\bar{F}_I)(\tilde{D}U + U - 1) - \frac{1}{4}\tilde{\chi}\tilde{D}U \\ &-32\mathrm{i}U\tilde{D}^2\Big[\Upsilon F_{\Upsilon\Upsilon} + \frac{1}{2}\mathrm{i}\Upsilon\,N^{IJ}(F_{I\Upsilon} - \bar{F}_{I\Upsilon})(F_{J\Upsilon} - \bar{F}_{J\Upsilon}) - \text{h.c.}\Big] \\ &-\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\Big[48U\tilde{D}^2 + 64\tilde{D}(U-1) - 2U\Upsilon + 32(U + U^{-1} - 2) - 8(1+U)\sqrt{-\Upsilon}\Big]\,, \end{aligned} \tag{3.12}$$

where we used the homogeneity of the function $F(Y,\Upsilon)$, which implies

$$F(Y,\Upsilon) = \frac{1}{2}Y^I F_I(Y,\Upsilon) + \Upsilon F_\Upsilon(Y,\Upsilon)\,. \tag{3.13}$$

To confirm that the entropy transforms as a function under electric-magnetic duality, one may make use of the results of [37]. Subsequently we require that $\mathcal{E}$ be stationary with respect to variations of $\tilde{D}$ and $\tilde{\chi}$. This imposes the conditions (we assume $U \neq 0$),

$$\begin{aligned} \tilde{D} &= 0\,, \\ \tilde{\chi} &= -\frac{16\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I\bar{F}_I) - 256\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)(1 - U^{-1}) \\ &\quad +32\mathrm{i}\sqrt{-\Upsilon}\Big[F_{I\Upsilon}N^{IJ}(\mathcal{Q}_J - \bar{F}_{JK}\mathcal{P}^K) - \text{h.c.}\Big]\,. \end{aligned} \tag{3.14}$$

Upon substitution of these equations into (3.12), the expression for $\mathcal{E}$ simplifies considerably and we obtain,

$$\begin{aligned}\mathcal{E}(Y,\bar{Y},\Upsilon,U) = {}& \frac{1}{2}U\,\Sigma(Y,\bar{Y},p,q) + \frac{1}{2}U\,N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L)\\ & - \frac{4\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I)(U-1)\\ & - \mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\Big[-2U\Upsilon + 32(U+U^{-1}-2) - 8(1+U)\sqrt{-\Upsilon}\Big]\,. \end{aligned} \tag{3.15}$$

Although this result is written in a different form and is obtained in a slightly different setting, it is in accord with the result derived in [25]. The entropy function (3.15) depends on the variables U, Υ and Y^I whose values will be determined at the attractor values where $\mathcal{E}$ is stationary. The macroscopic entropy is proportional to the entropy function taken at the attractor values,

$$\mathcal{S}_{\rm macro}(p,q) = 2\pi\mathcal{E}\Big|_{\rm attractor}\,. \tag{3.16}$$

In the following, we will discuss the extremization of $\mathcal{E}$ with respect to these variables, first in the absence of R^2-terms, and then for BPS black holes in the presence of R^2-terms. Finally we will consider the general case.

3.1 Variational equations without R^2-interactions

In the absence of R^2-interactions, the function F does not depend on Υ, so that the entropy function (3.15) reduces to

$$\begin{aligned}\mathcal{E}(Y,\bar{Y},\Upsilon,U) = {}& \frac{1}{2}U\,\Sigma(Y,\bar{Y},p,q) + \frac{1}{2}U\,N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L)\\ & - \frac{4\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I)(U-1)\,. \end{aligned} \tag{3.17}$$

Varying (3.17) with respect to Υ yields

$$U = 1\,. \tag{3.18}$$

The latter implies that the Ricci scalar of the four-dimensional space-time vanishes. Here we assumed that $\big(\bar{Y}^I F_I - Y^I \bar{F}_I\big)$ is non-vanishing, which is required so that Newton's constant remains finite. Varying with respect to U yields,

$$\Sigma + \big(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\big)\,N^{IJ}\,\big(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L\big) - \frac{8\mathrm{i}}{\sqrt{-\Upsilon}}\,\big(\bar{Y}^I F_I - Y^I \bar{F}_I\big) = 0\,, \tag{3.19}$$

which determines the value of Υ in terms of the Y^I. This relation is not surprising. When the function F depends exclusively on the Y^I, the quantity Υ is related to an auxiliary field in the original Lagrangian whose field equation is algebraic and (3.19) is a direct consequence of this equation.

Hence we are now dealing with an effective entropy function

$$\mathcal{E}(Y,\bar{Y},\Upsilon,1) = \frac{1}{2}\Sigma(Y,\bar{Y},p,q) + \frac{1}{2}N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L)\,, \tag{3.20}$$

which is independent of Υ, whose value is simply determined by (3.19). Note that (3.20) is homogeneous under uniform rescalings of the charges q_I and p^I and the variables Y^I. This implies that the entropy will be proportional the the square of the charges. Under infinitesimal changes of Y^I and $\bar{Y}^I$ the entropy function (3.20) changes according to

$$\begin{aligned}\delta\mathcal{E} = & \,\mathcal{P}^I\,\delta(F_I+\bar{F}_I) - \mathcal{Q}_I\,\delta(Y^I+\bar{Y}^I)\\ & +\frac{1}{2}\mathrm{i}\left(\mathcal{Q}_K - \bar{F}_{KM}\,\mathcal{P}^M\right)N^{KI}\,\delta F_{IJ}\,N^{JL}\left(\mathcal{Q}_L-\bar{F}_{LN}\,\mathcal{P}^N\right)\\ & -\frac{1}{2}\mathrm{i}\left(\mathcal{Q}_K - F_{KM}\,\mathcal{P}^M\right)N^{KI}\,\delta \bar{F}_{IJ}\,N^{JL}\left(\mathcal{Q}_L-F_{LN}\,\mathcal{P}^N\right) = 0\,,\end{aligned} \tag{3.21}$$

where $\delta F_I = F_{IJ}\,\delta Y^J$ and $\delta F_{IJ} = F_{IJK}\,\delta Y^K$. This equation determines the horizon value of the Y^I in terms of the black hole charges (p^I, q_I). Because the function $F(Y)$ is homogeneous of second degree, we have $F_{IJK}Y^K=0$. Using this relation one deduces from (3.21) that $\left(\mathcal{Q}_J - F_{JK}\,\mathcal{P}^K\right)Y^J=0$, which is equivalent to

$$\mathrm{i}(\bar{Y}^I F_I - Y^I\bar{F}_I) = p^I F_I - q_I Y^I\,. \tag{3.22}$$

Therefore, at the attractor point, we have

$$\Sigma = \mathrm{i}(\bar{Y}^I F_I - Y^I \bar{F}_I)\,. \tag{3.23}$$

Inserting this result into (3.19) yields

$$\sqrt{-\Upsilon} = \frac{8\,\Sigma}{\Sigma + N^{IJ}\left(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\right)\left(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L\right)}\,, \tag{3.24}$$

which gives the value of Υ in terms of the attractor values of the Y^I. Using (3.24) we can write the entropy as,

$$\mathcal{S}_{\text{macro}}(p,q) = 2\pi\,\mathcal{E}\Big|_{\text{attractor}} = \frac{8\pi\,\Sigma}{\sqrt{-\Upsilon}}\Big|_{\text{attractor}}\,. \tag{3.25}$$

Observe that, for a BPS black hole, $\mathcal{Q}_I = \mathcal{P}^J = 0$ and $\Upsilon = -64$, so that $\mathcal{S}_{\text{macro}} = \pi\,\Sigma|_{\text{attractor}}$ in accord with (2.29).

The entropy function (3.20) can be written as

$$\mathcal{E} = -q_I(Y^I+\bar{Y}^I) + p^I(F_I+\bar{F}_I) + \frac{1}{2}N^{IJ}(q_I - F_{IK}p^K)(q_J-\bar{F}_{JL}p^L) + N_{IJ}Y^I\bar{Y}^J\,, \tag{3.26}$$

where we used the homogeneity of the function $F(Y)$. Expressing the Y^I according to (2.23) (which is consistent with the first equation of (3.7), as we will show below) and using the definitions (2.21) and (2.22), we write (3.26) as follows,

$$\mathcal{E} = \frac{1}{2}\left(N^{IJ} + 2\,\mathrm{e}^{\mathcal{K}}X^I\bar{X}^J\right)(q_I - F_{IK}p^K)(q_J - \bar{F}_{JL}p^L)\,, \tag{3.27}$$

where F_{IJ} is now the second derivative of $F(X)$ with respect to X^I and X^J. Notice that this expression is invariant under uniform rescalings of the X^I by a complex number, which is a reflection of the complex scale invariance noted above (3.6).

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The quantities X^I can now be expressed in terms of the physical complex scalars belonging to the vector supermultiplets, which we denote by z^A, where the index A takes n values, one less than the number of vector fields. These scalars parametrize the special Kähler target space. Subsequently we parametrize the X^I as a projective holomorphic section (i.e. up to multiplication by a complex factor) in terms of the holomorphic coordinates z^A. We then use the identity (see the second reference in [37]),

$$N^{IJ} = \mathrm{e}^{\mathcal{K}(z,\bar{z})} \left[g^{A\bar{B}} \left(\partial_A + \partial_A \mathcal{K}(z,\bar{z})\right) X^I(z) \, \left(\partial_{\bar{B}} + \partial_{\bar{B}} \mathcal{K}(z,\bar{z})\right) \bar{X}^J(\bar{z}) - X^I(z)\, \bar{X}^J(\bar{z}) \right] \,, \tag{3.28}$$

where $g^{A\bar{B}}$ is the inverse metric of the special Kähler space, and write the entropy function (3.27) in the well-known form [3, 5],

$$\mathcal{E} = \frac{1}{2} \left[|Z(z,\bar{z})|^2 + g^{A\bar{B}}(z,\bar{z})\, \mathcal{D}_A Z(z,\bar{z})\, \mathcal{D}_{\bar{B}} \bar{Z}(z,\bar{z}) \right] \,, \tag{3.29}$$

where $\mathcal{D}_A Z = (\partial_A + \frac{1}{2}\partial_A \mathcal{K}) Z$. This agreement was also established in [40]. As mentioned above, in order to bring the entropy function into the form (3.29), we expressed the Y^I according to (2.23), which is consistent with the definition given in (3.7) by virtue of (3.22).

3.2 BPS black holes with R^2-interactions

In the presence of R^2 interactions, the horizon values of U and Υ for extremal BPS black holes are $U = 1$ and $\Upsilon = -64$ [11]. Inserting these values into (3.15) results in

$$\mathcal{E}(Y,\bar{Y},-64,1) = \frac{1}{2}\Sigma(Y,\bar{Y},p,q) + \frac{1}{2} N^{IJ} \left(\mathcal{Q}_I - F_{IK}\, \mathcal{P}^K\right) \left(\mathcal{Q}_J - \bar{F}_{JL}\, \mathcal{P}^L\right) \,. \tag{3.30}$$

Observe that the variational principle based on (3.30) is only consistent with the one based on (3.15) provided that (3.30) is supplemented by the extremization equations for U and for Υ given by (3.31) and (3.34) below. For BPS solutions it can be readily checked that the latter are indeed satisfied.

The form of the BPS entropy function (3.30) is closely related to the one given in [7, 12], which consists of the first term in (3.30). As discussed in section 2.2, the BPS attractor equations can be derived by a variational principle based on Σ. The quantity Σ was also used in [12] to construct a duality invariant version of the OSV integral which attempts to express microscopic state degeneracies in terms of macroscopic data [41]. In [12] it was furthermore shown that, for large BPS black holes, the evaluation in saddle-point approximation of the modified OSV integral precisely yields the macroscopic entropy (2.29). This result was established by computing the second variation of Σ which, upon imposing the BPS attractor equations $\mathcal{Q}_I = \mathcal{P}^J = 0$, equals $\delta^2\Sigma = 2N_{IJ}\,\delta Y^I\,\delta\bar{Y}^J$. Instead of constructing a duality invariant version of the OSV integral based on Σ, one can also consider constructing such an integral based on (3.30). The presence of the second term in (3.30) will, however, not affect the evaluation of this integral in saddle-point approximation (for large black holes), since, when evaluating the second variation of $\mathcal{E}$ on the BPS attractor, the second term contributes the same amount as the first term, so that $\delta^2\mathcal{E} = \delta^2\Sigma = 2N_{IJ}\,\delta Y^I\,\delta\bar{Y}^J$.

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3.3 Non-BPS black holes with R^2-interactions

In the following, we consider extremal black holes in the presence of R^2-terms and we compute the extremization equations for the fields U, Υ and Y^I following from the entropy function (3.15).

Varying with respect to U gives

$$\begin{aligned}
&\Sigma + \left(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\right) N^{IJ} \left(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L\right) - \frac{8\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I) \\
&-\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\Big[-4\Upsilon + 64(1-U^{-2}) - 16\sqrt{-\Upsilon}\Big] = 0\,. \qquad (3.31)
\end{aligned}$$

To verify the consistency with the analysis of the previous subsection (see (3.30)), one may verify that the BPS conditions $\mathcal{P}^I = \mathcal{Q}_I = 0$ and $\Upsilon = -64$ leaves only the term proportional to $(1-U^{-2})(F_\Upsilon - \bar{F}_\Upsilon)$ which vanishes as a result of $U = 1$.

Subsequently we consider the variation of the entropy function (3.15) with respect to arbitrary variations of the fields Y^I and Υ and their complex conjugates. Denoting this variation by $\delta = \delta Y^I \partial/\partial Y^I + \delta\bar{Y}^I \partial/\partial\bar{Y}^I + \delta\Upsilon\partial/\partial\Upsilon + \delta\bar{\Upsilon}\partial/\partial\bar{\Upsilon}$, we derive the following result,

$$\begin{aligned}
\delta\mathcal{E} =\; & U\left[\mathcal{P}^I\,\delta(F_I + \bar{F}_I) - \mathcal{Q}_I\,\delta(Y^I + \bar{Y}^I)\right] \\
& + \frac{1}{2}\mathrm{i}U\left[(\mathcal{Q}_K - \bar{F}_{KM}\,\mathcal{P}^M) N^{KI}\,\delta F_{IJ}\,N^{JL}(\mathcal{Q}_L - \bar{F}_{LN}\,\mathcal{P}^N) - \text{h.c.}\right] \\
& - 4\mathrm{i}(-\Upsilon)^{-1/2}\,(U-1)\,\left[(F_I - \bar{F}_I)\,\delta(Y^I + \bar{Y}^I) - (Y^I - \bar{Y}^I)\,\delta(F_I + \bar{F}_I)\right] \\
& + \mathrm{i}\left[2U\,\Upsilon - 32(U + U^{-1} - 2) + 16\sqrt{-\Upsilon}\right]\delta(F_\Upsilon - \bar{F}_\Upsilon) \\
& + \mathrm{i}U\left[\delta\Upsilon\,F_{\Upsilon I} N^{IJ}(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L) - \text{h.c.}\right] \\
& - 2\mathrm{i}(-\Upsilon)^{-3/2}\,(U-1)\,(\bar{Y}^I F_I - Y^I \bar{F}_I)\,\delta\Upsilon \\
& + \mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\left[U - 4(-\Upsilon)^{-1/2}\,(1+U)\right]\,\delta\Upsilon\,, \qquad (3.32)
\end{aligned}$$

where we took into account that the variable Υ is real.

Restricting ourselves to variations δY^I, the above result leads to the following attractor equations,

$$\begin{aligned}
& U\left(\mathcal{Q}_I - F_{IJ}\,\mathcal{P}^J\right) - \frac{1}{2}\mathrm{i}U\left(\mathcal{Q}_K - \bar{F}_{KM}\,\mathcal{P}^M\right) N^{KP}\,F_{PIQ}\,N^{QL}\left(\mathcal{Q}_L - \bar{F}_{LN}\,\mathcal{P}^N\right) \\
& + 4\mathrm{i}(-\Upsilon)^{-1/2}(U-1)\left[F_I - \bar{F}_I - F_{IJ}(Y^J - \bar{Y}^J)\right] \\
& - \mathrm{i}\left[2U\,\Upsilon - 32(U + U^{-1} - 2) + 16\sqrt{-\Upsilon}\right] F_{\Upsilon I} = 0\,. \qquad (3.33)
\end{aligned}$$

Upon variation of the entropy function with respect to Υ the resulting equation is only covariant with respect to electric/magnetic duality provided the attractor equations (3.33) are satisfied. However, one can apply a mixed derivative of the form $\delta = \partial/\partial_\Upsilon + \mathrm{i}F_{\Upsilon I}\,N^{IJ}\,\partial/\partial Y^I$, which has the property that when acting on a symplectic function $G(Y,\Upsilon)$, then also δG transforms as a symplectic function [37]. An alternative derivation is based on $\delta = Y^I\partial/\partial Y^I + \bar{Y}^I\partial/\partial\bar{Y}^I + 2\Upsilon\partial/\partial\Upsilon$, using that Υ is real so that

$\partial/\partial\Upsilon$ acts on both Υ and $\bar{\Upsilon}$. Exploiting the homogeneity properties of the various quantities involved, one derives the equation,

$$\begin{aligned} & U\Sigma - \mathrm{i}(\bar{Y}^I F_I - Y^I \bar{F}_I)\Big[U + 4(-\Upsilon)^{-1/2}(U-1)\Big] \\ & +2\mathrm{i}U\left[\Upsilon F_{I\Upsilon} N^{IJ}(\mathcal{Q}_J - \bar{F}_{JK}\mathcal{P}^K) - \text{h.c.}\right] \\ & +2\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\Big[2U\Upsilon + 4\sqrt{-\Upsilon}(1+U)\Big] = 0\,. \end{aligned} \tag{3.34}$$

Note that the above equations (3.33) and (3.34) are indeed satisfied in the BPS case. They are also consistent with electric/magnetic duality.

4. Discussion

In this paper we studied the entropy function for static extremal black holes using the proposal of [13] and we exhibited its relation with the entropy function for BPS black holes in $N=2$ supergravity, derived in [12]. For BPS black holes these two entropy functions lead to the same results for the attractor equations and the entropy. This result even persists in the semi-classical approximation when evaluating an inverse Laplace integral of the OSV-type [12].

In this final section we would like to discuss two more issues. The first one deals with the presence of higher-derivative couplings other than those introduced in section 3. The latter are associated with interactions quadratic in the Riemann tensor and are encoded by the Υ dependence in the holomorphic function $F(Y,\Upsilon)$. Take, for instance, the simple example based on

$$F(Y,\Upsilon) = -\frac{Y^1Y^2Y^3}{Y^0} - C\,\frac{Y^1}{Y^0}\,\Upsilon\,. \tag{4.1}$$

For BPS black holes the attractor equations can be solved for generic charges [10], but solutions only are only consistent when the charges satisfy certain relations in which case one obtains an explicit expression for the entropy. These relations are not satisfied when the black hole carries the following non-vanishing charges,

$$q_0 = p^1 = Q\,, \qquad p^2 = p^3 = P\,, \tag{4.2}$$

with PQ positive. However, in that case [25], non-supersymmetric black holes are possible and one can attempt to solve the equations (3.31), (3.33) and (3.34). Unfortunately explicit solutions do not exist and one has to resort to perturbation theory in the constant C. To first order in C, the attractor values read,

$$\begin{aligned} Y^0 &= \tfrac{1}{4}P\left(1 + 96\,C\,P^{-2}\right)\,, \\ Y^1 &= \tfrac{1}{4}\mathrm{i}\,Q\left(1 + 40\,C\,P^{-2}\right)\,, & U &= 1 - 16\,C\,P^{-2}\,, \\ Y^2 &= Y^3 = \tfrac{1}{4}\mathrm{i}\,P\left(1 + 16\,C\,P^{-2}\right)\,, & \Upsilon &= -4\,. \end{aligned} \tag{4.3}$$

In this order of perturbation theory the corresponding entropy (3.16) is computed by substituting the tree-level values for U, Υ and the Y^I into the entropy function (3.15). The result reads,

$$\mathcal{S}_{\text{macro}} = 2\pi PQ\left(1 + 40\,C\,P^{-2}\right)\,. \tag{4.4}$$

As was argued in [25] this is not the expected value from microstate counting [42, 43], which requires a different numerical factor in front of the CP^{-2} correction term. However, one has to take into account that other higher-derivative interactions may be present, associated with matter multiplets instead, which would in principle contribute to the entropy. Such higher-derivative interactions have been studied for $N = 2$ tensor supermultiplets, and, indeed, it turns out that they lead to entropy corrections for non-supersymmetric black holes [44]. For BPS black holes, however, these corrections vanish. Although a comprehensive treatment of higher-derivative interactions is yet to be given for $N = 2$ supergravity, it seems that this result is generic.

These observations are in line with more recent findings [27, 28] based on heterotic string α'-corrections encoded in a higher-derivative effective action in higher dimensions, which lead to additional matter-coupled higher-derivative interactions in four dimensions. When these are taken into account, the matching of the macroscopic entropy with the microscopic result is established [28].

A second topic concerns possible non-holomorphic corrections to the results presented in section 3. The Lagrangian (3.1) is based on a holomorphic homogeneous function $F(X, \hat{A})$, which subsequently is written in terms of the variables Y^I and Υ, and corresponds to the so-called effective Wilsonian action. This action is based on integrating out the massive degrees of freedom and it describes the correct physics for energy scales between appropriately chosen infrared and ultraviolet cutoffs. In order to preserve physical symmetries non-holomorphic contributions should be included associated with integrating out massless degrees of freedom. In the special case of heterotic black holes in $N = 4$ supersymmetric compactifications, the requirement of explicit S-duality invariance of the entropy and the attractor equations allows one to determine the contribution from these non-holomorphic corrections, as was first demonstrated in [10] for BPS black holes. In [45, 12] it was established that non-holomorphic corrections to the BPS entropy function (2.25) can be encoded into a real function $\Omega(Y, \bar{Y}, \Upsilon, \bar{\Upsilon})$ which is homogeneous of second degree. The modifications to the entropy function are then effected by substituting $F(Y, \Upsilon) \rightarrow F(Y, \Upsilon) + 2\mathrm{i}\Omega(Y, \bar{Y}, \Upsilon, \bar{\Upsilon})$. There are good reasons to expect that this same substitution should be applied to the more general entropy function (3.15). Indeed, when applying this ansatz to heterotic black holes in $N = 4$ supersymmetric compactifications, the resulting entropy function is S-duality invariant and can be used to analyze non-supersymmetric extremal black holes in the same way as was done for the BPS black holes. In that case $\partial_\Upsilon(F + 2\mathrm{i}\Omega)$ has to be an S-duality invariant function.

Unlike the BPS entropy function (2.25), the entropy function (3.15) was derived directly from an effective action. Hence one may reconsider the relevant parts of this effective action given in (3.5), in order to see whether additional changes beyond the substitution $F(Y, \Upsilon) \rightarrow F(Y, \Upsilon) + 2\mathrm{i}\Omega(Y, \bar{Y}, \Upsilon, \bar{\Upsilon})$ are needed in order to reproduce the conjectured non-holomorphic modification of the entropy function. As it turns out only one minor change is required. Namely, one has to replace the coefficient $(F - F_I X^I + \frac{1}{2}\bar{F}_{IJ} X^I X^J)$ of the $(T_{abij}\varepsilon^{ij})^2$ term in $\mathcal{L}_2$ by $(\hat{A}F_A - \frac{1}{2}F_I X^I + \frac{1}{2}\bar{F}_{IJ} X^I X^J)$. For a holomorphic function $F(X, \hat{A})$ these two expressions coincide by virtue of (3.13), but when the non-holomorphic function $\Omega(X, \bar{X}, \hat{A}, \bar{\hat{A}})$ is included, the two expressions will be different. Of course, the presence of non-holomorphic terms will affect the supersymmetry of the original action. Since the non-holomorphic corrections are expected to capture the contributions of the massless modes, one expects that their supersymmetrization will contain non-local interactions. The construction of such a supersymmetric action is a challenge.

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